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Medium damping influences on the resonant frequency and quality factor of piezoelectric circular microdiaphragm sensors

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Abstract
Medium damping influences on the resonant frequency and quality factor of piezoelectric circular microdiaphragm sensors (PCMSs) are investigated theoretically and experimentally in this paper. The acoustic radiation and viscosity damping as the two main sources of energy dissipation in a medium virtually added the mass of the diaphragm and therefore decrease the frequency and $Q$-factor of the diaphragm. The magnitude of medium damping inversely depends on the radius-to-thickness ratio. An increase in this ratio is the trend in the fabrication of thin microdiaphragms by MEMS fabrication processes, which implies the higher influence of medium damping on the dynamic behavior of microdiaphragms. The fabricated PCMSs were tested in vacuum, air, and ethanol. The $Q$-factor and the resonant frequency of the device increase by almost seven times, 4.7% from air to 0.05 atm pressure, respectively. The $Q$-value drops from 111.195 in air to 23.908 in ethanol. Throughout this work, theoretical and experimental values were compared and a fairly good correlation was observed.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Micro- or nanoelectromechanical resonant sensors are devices whose resonant frequency shifts as a function of a physical or chemical parameter [1–6]. They have been used extensively as highly sensitive sensors with different actuation mechanisms, such as optical, electrostatic, electromagnetic, and piezoelectric for detecting gases, chemicals, or biological entities [7–10]. Modeling these sensors regardless of their shapes and applications as a simple one-dimensional damped harmonic oscillator will result in the familiar equation of motion [11],

$$\ddot{x}(t) + \frac{\omega_0}{Q} \dot{x}(t) + \omega_0^2 x(t) = \frac{f(t)}{m_{eff}}.$$ (1)

Here, $f(t)$ is the driving force, $m_{eff}$ is the effective mass, and $Q$ is the quality factor of the mode in question. For mass sensing, it is desirable to minimize $m_{eff}/Q$. In vacuum, $m_{eff}$ is the mass of the resonator itself; however, for a resonator vibrating in a fluid, the mass of surrounding fluid affects this mass, which in turn reduces the resonant frequency of the sensor and degrades the mass sensitivity. The sensor $Q$-factor is defined as $Q = 2\pi W_0/\Delta W$, where $W_0$ is the stored vibrational energy and $\Delta W$ is the total energy lost per cycle of vibration. The energy lost term can be written as $\Delta W = \sum \Delta W_i$, where $\Delta W_i$ represents the different dissipation mechanisms that contribute to the total energy loss. Overall, the total $Q$-factor ($Q_{tot}$) of a device can be written as [12]

$$\frac{1}{Q_{tot}} = \frac{1}{Q_{medium}} + \frac{1}{Q_{clamping}} + \frac{1}{Q_{bulk}} + \frac{1}{Q_{others}}.$$ (2)

The main energy dissipation mechanisms for a mechanical resonator are identified as (a) medium loss, which is the loss into the surrounding (fluid) medium due to acoustic radiation [13] or viscous drag [14], (b) clamping or support loss, which is the dissipation of energy through the support used to mount the resonator results from the vibration of the resonator [15, 16], and (c) bulk loss, which is composed of a variety of...
physical mechanisms, such as internal friction, thermoelastic dissipation (TED), phonon–phonon scattering, motion of lattice defects, and piezoelectric damping in piezoelectric materials [12, 17].

These energy dissipation mechanisms do not equally contribute to the total energy loss of the system. Size and ambient pressure are the two main parameters, which clarify the contribution of these terms. Size reduction from macro to micro scale increases the resonator’s surface-to-volume ratio, and hence signifies the effect of surface forces, and dominates them over the body forces. Therefore, bulk losses are negligible in microscale regions compared to medium damping terms. For a resonator vibrating in air, the medium damping is heavily dependent on the ambient pressure [18]. Damping terms. For a resonator vibrating in air, the medium damping is dominant. In a high vacuum region due to the elimination of surrounding air, the bulk losses regain their significance and become the dominant dissipation mechanism. In between, there is a molecular region where the surface dissipation due to the independent collision of non-interacting air molecules with the moving surface of the resonator is the dominant loss.

Based on the aforementioned notes, the dominant damping mechanism for piezoelectric circular microdiaphragm sensors (PCMSs) working in normal atmosphere or in aqueous environment is the medium damping. Therefore, in this work, we theoretically and experimentally investigate influences of this damping composed of acoustic radiation and viscous terms, in the resonant frequency and quality factor of PCMSs.

2. Theory

2.1. Plate vibrating in vacuum

Let us consider a thin circular plate made of a linear elastic, homogeneous and isotropic material having radius \( a \), mass density \( \rho_p \), and thickness \( h \). The plate is clamped around its edges and vibrates in vacuum. Moreover, the effects of shear deformation and rotary inertia are neglected. Lamb [13] in 1920 approximated the normal displacement profile of this plate by an assumed mode shape,

\[
w(r, t) = X(t) \left(1 - \frac{r^2}{a^2}\right)^2,
\]

where \( w(r, t) \) is the transverse displacement, and \( X(t) \) is a function of time. It is worth noting that the exact mode shape of the plate vibrating in vacuum is a combination of Bessel functions, as shown in equation (4) [20]; however, it was previously proved that the assumed mode shape is an adequate approximation [21],

\[
w(r, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left[ A_{mn} J_m(\lambda_{mn} r/a) + B_{mn} I_m(\lambda_{mn} r/a) \right] \cos m(\theta - \varphi_m) e^{i\omega t}.
\]

The maximal kinetic and potential energies of the diaphragm vibrating in vacuum are

\[
T_p = \frac{\pi \rho_p a h^2}{10} u_0^2, \quad V_p = \frac{8\pi E h^3}{9(1 - \nu^2) a^2} X^2,
\]

where \( E \) is Young’s modulus, \( u_0 = (dX/dr)_{\text{max}} \) and \( \nu \) is Poisson’s ratio. Applying the Rayleigh–Ritz method, the first resonant frequency of the diaphragm is obtained as

\[
f_{\text{vac}} = 0.4745 \frac{hc_p}{a^2}, \quad c_p = \sqrt{\frac{E_p}{(1 - \nu^2)h_p}}.
\]

2.2. Plate in contact with an inviscid fluid (acoustic radiation damping)

For a plate in contact with an inviscid and incompressible fluid with density \( \rho_f \) on one side, the presence of the fluid has the effect of lowering the frequency on account of the increased inertia, and damping of the vibrations owing to the energy carried off in the form of sound waves. Therefore, this type of damping is known as acoustic radiation or added mass effect. The kinetic energy of the fluid in contact with the plate was expressed by Lamb [13] as follows,

\[
T_f = 0.2102 \rho_f a^3 u_0^2.
\]

The effect of fluid is virtually to increase the inertia in the ratio of \( \beta \), where \( \beta \) is known as the added virtual mass factor,

\[
\frac{T_p + T_f}{T_p} = 1 + \beta, \quad \beta = 0.6689 \frac{\rho_f a}{\rho_p h}.
\]

Therefore, the resonant frequency of the plate in the fluid is lowered by a factor of

\[
f_f = \frac{f_{\text{vac}}}{\sqrt{1 + \beta}}.
\]

The rate of damping is estimated by calculating the emitted energy in the form of sound waves into the fluid [13], and it is obtained as

\[
\alpha = \frac{5\pi^2 \rho_f}{9} \frac{f_f^2 a^2}{(1 + \beta) c_f}.
\]

Hence, the \( Q \)-factor of acoustic radiation is

\[
Q = \frac{\pi f_f}{\alpha} = 1.20 \frac{\rho_p c_f}{\rho_f c_p} (1 + \beta)^{1.5}.
\]

2.3. Plate in contact with a Newtonian fluid (acoustic radiation and viscous damping)

It was mentioned earlier that besides the acoustic radiation term, viscous damping is also a significant part of energy dissipation in microsystems. The viscous damping can be divided into two parts: damping in free space (also called drag force damping) [14] and squeeze film damping, which occurs in narrow gaps [22, 23]. The squeeze film damping arises when the membrane vibrates in parallel with a wall. The air film between the plate and the wall is squeezed so that some of the air flows in and out of the gap. This flow dissipates the vibrational energy of the membrane and therefore acts as a
damper. Obviously, this damping depends on the gap between the diaphragm and the wall. When the plate is very far away from the wall, the damping force is reduced to the drag force only. In our case, the gap is at least two times the diaphragm radius and therefore the squeeze film term can be neglected [22].

Viscous damping is exerted on the device because of friction between the fluid and the surface of the resonator. The fluid flow around the resonator is described by the Navier–Stokes equation. The simplified Navier–Stokes equation for incompressible flow with constant viscosity is

\[ \rho_f (v \cdot \nabla) v + \rho_f \frac{\partial v}{\partial t} = \mu \Delta v - \nabla p, \]

where \( p \), \( \mu \), and \( v \) are the pressure, dynamic viscosity, and velocity, respectively. This equation cannot be solved analytically, and some approximations should first be introduced. Kozlovsky [24] employed the stream function method [25] to analytically calculate the medium damping. He defined a scalar stream function \( \psi(r, z) \), from which the velocity is derived,

\[ v_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}, \quad v_z = \frac{1}{r} \frac{\partial \psi}{\partial r}. \]

The advantage of this approach is that the continuity equation of an incompressible fluid is automatically satisfied,

\[ \frac{1}{r} \frac{\partial}{\partial r} (rv_r) + \frac{\partial}{\partial z} v_z = 0. \]

The vorticity \( \omega = \nabla \times V \) in axisymmetrical flow has only one component in the \( \theta \) direction, which is denoted by \( \Omega \). The relation between the vorticity and the stream function is written as

\[ \Omega = \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 \psi}{\partial z^2}. \]

Rewriting the Navier–Stokes equation with the vorticity term,

\[ \frac{\partial}{\partial t} \omega = \nabla \cdot (\omega \otimes V) + v \nabla^2 \omega , \]

where \( \nu = \mu/\rho \) is the kinematic viscosity. The nonlinear terms in the square brackets are negligible to the diffusion term in the case of small plate velocity \( (u_0 \ll \nu/A) \). They are also negligible compared to the time-derivative term if \( u_0 \ll \omega a \). Denoting the plate velocity amplitude as \( u_0 = \omega A \), where \( A \) is the amplitude of the vertical displacement of the plate, and recalling that plate elastic theory requires that \( A \ll a \), the condition is verified. By neglecting those nonlinear terms and solving the new linear equation, the vorticity \( \Omega \) and the stream function \( \psi \) are obtained. Subsequently, the kinetic energy of the fluid surrounding the diaphragm is obtained as [24]

\[ T_f = \rho_a^0 u_0^3 \left( 0.06689 \pi + \frac{3}{10} \sqrt{2} \xi + O(\xi^3) \right), \]

where \( \xi \) is a nondimensional parameter,

\[ \xi = \frac{\nu}{\omega a \rho_a^0} . \]

With recruiting equation (8), the added virtual mass due to both acoustic radiation and viscosity term is

\[ \beta = 0.6689 \frac{\rho_f}{\rho_p a^3} (1 + 1.057 \xi + O(\xi^3)) . \]
3.2. Measurement procedure

The dynamic behavior of the PCMSs in a different medium was first examined by measuring their resonant frequency in vacuum and then subsequently in air and liquid. An Agilent 4294A impedance analyzer was used to characterize the resonant frequency behavior of the PCMS. The diaphragms were excited by 100 mV ac voltage. The frequency spectrum was divided by 801 sample points, and the measurement bandwidth was adjusted to the highest precision of the instrument. In vacuum, the frequency was recorded in different ambient pressures using a vacuum chamber. The pressure in the chamber was monitored using a pressure gauge and is controlled by a flow rate valve. For this measurement, a chip containing an array of several diaphragms was packaged by a patterned PMMA sheet with spacers between the package and the testing floor to prevent the diaphragms from being sealed. Figure 3 shows the setup of this experiment. Using this setup, resonance measurements were carried out at chamber pressures ranging from 1 to 0.05 atm.

In order to study the behavior of the PCMS in ethanol, the fluidic cell was connected through capillary tubes (1 mm inner diameter) and adaptive ports to a syringe pump. The fluidic cell consisted of two patterned PMMA sheets thermally bonded to each other, and the PCMS glued to the PMMA sheet. This setup, shown in figure 4, allows the liquid to continuously stream across the piezoelectric membranes with a constant velocity fixed by the syringe pump.

4. Results and discussion

4.1. Device characterization

An example of impedance and phase response of a PCMS is shown in figure 5. PCMSs always work in their dynamic modes, which means that they vibrate at their resonant frequency. For instance, when a PCMS is applied as a biosensor, the sensor is first excited electrically, so that it vibrates at its resonance frequency. Once a biological entity is captured by a sensor, the working resonant frequency will change. The captured mass can be measured by the detection of this frequency depression. The measured resonance frequency \( f_r \) and the anti-resonance frequency \( f_a \) from the impedance spectrum are used to determine the effective coupling coefficient of the diaphragm [27]. The coupling factor \( k^2 \) directly influences the efficiency of power emission, bandwidth, and sensitivity of the sensor [28]. For piezoelectric materials, the coupling coefficient \( k^2 \) can be defined as the ratio of the stored mechanical energy in the piezoelectric material to the input electrical energy supplied by an electrical source. For instance, the effective electromechanical coupling coefficient \( k^2 \) of the fabricated PCMS, shown in figure 5, was 1.27%. In the experimental part of this work, the \( Q \) value is measured by the relation \( \frac{f_0}{\Delta f} \), where \( f_0 \) is the frequency at which the real part of the impedance reaches its maximum, and \( \Delta f \) is the width of the peak at its half height. \( f_0 \) and \( \Delta f \) are estimated by fitting the measured phase peak using the Lorentz function [6]. For instance, the \( Q \)-factor of the PCMS shown in figure 5 was 73.125 at a quite low operating frequency of around 107.202 kHz. This value is comparable to that of a typical FBAR (film bulk acoustic resonators) operating at around GHz [7], which means that both of them have a similar resolution in detecting frequency shift.
4.2. Resonant frequency behavior

We tested the frequency response of the PCMS at different ambient pressures from normal atmosphere (1 atm = 760 torr) to 0.05 atm. The results of these measurements are shown in figure 6. As was expected, the resonant frequency of the diaphragm decreases from vacuum to normal atmosphere due to an added mass effect. We observed a frequency change of around 4.7% from air to 0.05 atm pressure. At lower pressures, the peaks also demonstrate nonlinear behavior in their vibration. This nonlinearity, which was fully explained in our previous paper [26], originally comes from a change in the restoring force, such as the flexural rigidity or membrane tension, due to large vibration amplitude [29]. In vacuum due to the elimination of the added mass effect, the vibration amplitude increases, and therefore the nonlinearity in vibration of the PCMS was observed.

We employed equation (9) to calculate the frequency shift of the sensor at each different pressure. In these calculations, the material properties of different layers and the air properties listed in tables 1 and 2 were used. The average density of the diaphragm and the total thickness of the plate are \( \rho_p = \sum \rho_i h_i / h_p = 5548 \text{ kg m}^{-3} \) and \( h_p = 3.65 \text{ } \mu\text{m} \), respectively. The density of air is pressure dependent and it was calculated by

\[
\rho_f = 1.18 \times 10^{-5} p / p_0.
\]

where \( p_0 = 1 \text{ Pa} \) and \( p \) is the ambient pressure. The value of the velocity of sound in air, which is independent of pressure, is shown in table 1. The comparison between the theoretical and experimental values of resonant frequency in different pressures is summarized in table 3. We choose the frequency of the diaphragm at 0.05 atm as the resonant frequency in vacuum \( (f_{\text{vac}} = 58.045 \text{ kHz}) \), and the subsequent values of frequencies calculated based on this frequency. Table 3 clearly demonstrates good agreement between the proposed theoretical values by the Lamb method and the experiment.

We theoretically investigated the frequency behavior of a PCMS in different media for a radius range \( (300 \text{ } \mu\text{m} \leq r \leq 325 \text{ } \mu\text{m}) \), and the results are summarized in figure 7. For
Table 2. Air and ethanol properties.

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Dynamic viscosity $\mu$ (Pa s)</th>
<th>Density $\rho$ (kg m$^{-3}$)</th>
<th>Kinetic viscosity $\nu$ (m$^2$ s$^{-1}$)</th>
<th>Speed of sound $c_f$ (m s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>$1.8 \times 10^{-5}$</td>
<td>1.18</td>
<td>$1.53 \times 10^{-5}$</td>
<td>343</td>
</tr>
<tr>
<td>Ethanol</td>
<td>$1.2 \times 10^{-3}$</td>
<td>789</td>
<td>$1.52 \times 10^{-6}$</td>
<td>1144</td>
</tr>
</tbody>
</table>

Table 3. Comparison of the theoretical and experimental frequency at different pressures.

<table>
<thead>
<tr>
<th>Pressure</th>
<th>$f_{\text{theor}}$ (error%)</th>
<th>$f_{\text{exp}}$</th>
<th>Pressure</th>
<th>$f_{\text{theor}}$ (error%)</th>
<th>$f_{\text{exp}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 atm</td>
<td>58.039 (1.3)</td>
<td>57.309</td>
<td>0.6 atm</td>
<td>58.011 (3.5)</td>
<td>56.071</td>
</tr>
<tr>
<td>0.2 atm</td>
<td>58.033 (1.9)</td>
<td>56.958</td>
<td>0.7 atm</td>
<td>58.005 (3.9)</td>
<td>55.838</td>
</tr>
<tr>
<td>0.3 atm</td>
<td>58.028 (2.2)</td>
<td>56.774</td>
<td>0.8 atm</td>
<td>58.000 (4.2)</td>
<td>55.687</td>
</tr>
<tr>
<td>0.4 atm</td>
<td>58.022 (2.6)</td>
<td>56.522</td>
<td>0.9 atm</td>
<td>57.994 (4.4)</td>
<td>55.536</td>
</tr>
<tr>
<td>0.5 atm</td>
<td>58.017 (3.1)</td>
<td>56.255</td>
<td>1 atm (air)</td>
<td>57.988 (4.8)</td>
<td>55.319</td>
</tr>
</tbody>
</table>

Figure 7. Frequency of a PCMS in a radius range working in a different medium.

radius = 300 $\mu$m, the diaphragm frequency decreases by almost 0.58% from vacuum to air. When the diaphragm vibrates in ethanol, Lamb (inviscid fluid) and Kozlovsky (viscous fluid) models estimate the frequency shift of around 66.32% and 66.63% from the vacuum condition. The small difference in the values predicted by these two models is due to the low kinetic viscosity of ethanol ($\nu = 1.2 \times 10^{-6}$ m$^2$ s$^{-1}$). For higher viscosities, for instance, 10 or 100 times of the ethanol viscosity, this frequency shift is 67.27% and 69.07%, respectively. (For having a sense of viscosity values, the blood viscosity is $\nu = 3-4 \times 10^{-6}$ m$^2$ s$^{-1}$, and the olive oil viscosity is $\nu = 9 \times 10^{-5}$ m$^2$ s$^{-1}$.)

In order to compare the ratio of contribution of the viscosity to the acoustic radiation terms on the frequency shift of a PCMS, figure 8 was drawn. In this figure, the ratio of the viscous term to the acoustic radiation term ($\beta_{\nu \alpha}/\beta_{\text{ar}} = 1.057\%$) is plotted over kinematic viscosity. The vertical axis in this figure, as stated earlier, is proportional to the nondimensional parameter $\xi$:

$$\xi = \frac{\nu}{\omega a^2}, \quad \omega \propto \frac{hc_p}{a^2}.$$  

By replacing the natural frequency $\omega$ in $\xi$, it was found that the viscosity effect is mainly dependent on ($\nu/hc_p)^{0.5}$. This means that, besides the kinematic viscosity, the thickness and the sound velocity in the plate are also two significant parameters in viscosity influences on the frequency shift. For instance, for constant sound velocity and viscosity, the influence of the viscous term increases by decreasing the thickness $h$. This is the trend in microfabrication technology to reduce the sizes, which shows that the viscosity influences gain higher importance in lower sizes. For our fabricated PCMS, the velocity of sound in the diaphragm was $c_p = 5277$ m s$^{-1}$, which was calculated by material properties listed in table 1. Figure 8 demonstrates that the viscosity influences reach 4.8% in the viscosity of hundred times of ethanol; however, in ethanol range viscosity, this contribution can be neglected. This is in agreement with the experimental observations of Ayela and Nicu [30] that only high viscosities (higher than 10 cp) have a significant influence on the frequency shift of the piezoelectric sensor.

Figure 8. Relative contribution of viscosity to acoustic radiation term on frequency as a function of kinematic viscosity.
4.3. Q-factor analysis

The discussion so far has focused on the frequency analysis of a PCMS as a function of the surrounding fluid. However, as the effect of the added mass and liquid viscosity on the behavior of the PCMS is our primary interest in this study, it would be worth studying these parameters on the Q-factor values. As mentioned earlier, the Q-factor in the experimental part is defined as \( Q = f_0/\Delta f \), where the values of \( f_0 \) and \( \Delta f \) are obtained by fitting the measured phase peak using the Lorentz function. Figure 9 demonstrates the calculated Q values of the frequency peaks shown in figure 6. As expected, at lower pressures, the air damping decreases and therefore the Q-factor increases. Experimental results show that the Q-factor of the PCMS in 0.05 atm is around seven times higher than the sensor working at normal atmosphere. We also theoretically calculate the Q-factor of the device in air using equations (11) and (21). The calculation demonstrates that for a diaphragm with radius \( a = 500 \mu m \), \( Q_{ar} \) and \( Q_{vis} \) are 377 and 2979, respectively. This result clarifies that the viscosity of the air does not play an important role in the Qtot of the diaphragm that vibrates in the air.

The theoretical \( Q_{tot} \) of the PCMS in air for a radius range of \( 300 \mu m \leq r \leq 700 \mu m \) is illustrated in figure 10. This curve is an upper bound of the device Q-factor in air, because other damping sources, such as intrinsic or support damping, were neglected in the theoretical section. The Q-value for this radius range was between 324 and 346. The highest measured experimental value of the Q-factor was 137. The Q-factors of nine different samples are depicted in figure 10. The Q-factors vary from device to device, even for the same radius. The reason is that the Q-factor is dependent on the physical and chemical quality of the piezoelectric layer, the thickness of layers, and stress issues in different layers, which may vary case by case. The measured Q-values define a rectangular region that confines the Q-values between the highest and the lowest Q-factor measured experimentally.

The Q-factor of a PCMS was also investigated in ethanol. Figure 11 shows the phase response of a PCMS in air and in ethanol. The Q-factor of that device in air was 111.195, and this amount decreases to 23.908 in ethanol. This is the 4.687 times reduction in the Q-value. It should be underlined that this Q-factor is higher than the Q-factors of most of state-of-the-art micromachined cantilevers used for specific applications in liquid media. The higher Q value in the liquid is the main advantage of microdiaphragms over microcantilevers [31]. The theoretical calculation of the Q-factor in ethanol for a diaphragm with \( a = 400 \mu m \) results in \( Q_{ar} = 70.64 \), \( Q_{vis} = 123.53 \), and therefore \( Q_{tot} = 44.94 \). It is worth emphasizing that the theoretical Q-factor calculation in this work is only an estimation of the total Q of the device. Therefore the large difference in Q-values is reasonable.
Figure 12. The contribution of viscosity and acoustic radiation terms on the $Q$-value of a PCMS in a radius range from 300 to 700 $\mu$m.

Figure 12 demonstrates the contribution of viscosity and acoustic radiation to the $Q$-factor of a PCMS with radius 300 $\mu$m $\leq r \leq$ 700 $\mu$m working in ethanol. For this radius range, the added virtual mass factor changes in the range of 7.88 $\leq \beta \leq$ 18.43. This value is between 0.012 and 0.028 for a diaphragm working in air. The figure demonstrates that the predominant parameter in $Q_{\text{tot}}$ is the acoustic radiation term. $Q_{ar}$ is between 47.90 and 154.42, while $Q_{\text{vis}}$ is between 35.40 and 80.36. The $Q_{\text{tot}}$ value is between 35.40 and 80.36. The figure also shows that $Q_{\text{medium}}$ increases in larger radii, which implies that the medium influences have less significance in $Q_{\text{tot}}$ of the device in bigger sizes. This in accordance with experimental observations that in the macro scale region the intrinsic dissipation mechanisms are the predominant factor in the calculation of the $Q$-factor of the resonating device.

5. Conclusion

Piezoelectric circular microdiaphragm sensors (PCMS) were fabricated by combining a sol–gel PZT thin film and MEMS technology. Their dynamic behavior under medium damping was fully investigated. The contribution of viscosity over acoustic radiation damping is inversely related to the thickness of the diaphragm. Hence, the viscosity has more significance in thinner diaphragms. It was shown that the viscosity influences on frequency reach 4.8% in the viscosity of hundred times of ethanol; however, in ethanol range viscosity this contribution can be neglected. The added virtual mass factor ($\beta$) varies from 0.028 in air to 18.43 in ethanol for a PCMS with radius $a = 700$ $\mu$m, which clearly shows the significant effect of the medium on the frequency shift of the microdiaphragm. A $Q$-factor as high as 137 was measured in air. Theoretical calculations of $Q$ for a PCMS with radius $a = 400$ $\mu$m in ethanol result in $Q_{ar} = 70.64$, $Q_{\text{vis}} = 123.53$, and therefore $Q_{\text{tot}} = 44.94$. These values are higher than the $Q$-factor of most state-of-the-art micromachined cantilevers. This high $Q$ value in liquid is the main advantage of microdiaphragms over microcantilevers.

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